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Study of Dynamical Behavior of Cosmological Constant *Λ* and Aspects of Phenomenological Models From Ia Supernovae

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Abstract: In this paper, we have solved the Einstein field equation for the scalar factor R(t), density ρ(t) and cosmological term $\Lambda(t)$, by assuming $\Lambda = \mu H^2$, where μ is a parameter of $\Lambda \sim H^2$ in the context of Higher dimension space time. It has been shown that model $\Lambda \sim H^2$ is equivalent to $\Lambda \sim \left(\frac{\dot{R}}{R}\right)^2$, $\Lambda \sim \left(\frac{\ddot{R}}{R}\right)$, and $\Lambda \sim \rho$ models, when the condition for R(t), ρ(t), and Λ(t) are expressed in terms of Ω_m and Ω_Λ, the matter and vacuum energy density of the universe respectively.

Key words: general relativity, cosmological parameters, phenomenological models

1. Introduction

Recent years have witness resurgence of interest in the possibility that a positive Λ-term (a cosmological constant) may dominate the total energy density in the universe. The observations on magnitude and redshift of type Ia Supernovae made, independently, Perlmutter et. al. (1999) and Riess et. al. (1998) appear to suggest that our universe may be accelerating with a large fraction of the cosmological density in the form of cosmological Λ-term.

On the other hand, models with a dynamic cosmological term Λ(t) has been considered in numerous papers to explain the observed small value Λ, which is about 120 orders of magnitude below the value for the vacuum energy density predicted by quantum field theory (Wienberg 1989, Carroll et. al. 1992, Sahni and Starobinsky 2000). It has been argued that, due to the coupling of the dynamic degree of freedom with the matter fields of the universe, Λ relaxes to its present small value through the expansion of the universe and the creation of photons. This approach is essentially phenomenological in nature but explains, in a natural way, the present small value of Λ which might be large in the early universe. From this point of view, the cosmological constant is small because the universe is old.

As the dynamics of the variable Λ-models depends sensitively on the chosen dynamic law for the variation of Λ and, in general, becomes altogether different from the dynamics of corresponding constant Λ-models, there is no reason to believe that the observations distant objects would also agree with the variable Λ-models given that they agree with the corresponding constant Λ-ones, especially for the same estimates of the parameters. In this view, it would be worthwhile to test the consistency of these observations with the variable Λ-models and find the estimates of different cosmological parameters required by these models.

As a consequence of the search for current status of this acceleration some phenomenological models of Λ, viz. $\Lambda \sim \left(\frac{\dot{R}}{R}\right)^2$, $\Lambda \sim \left(\frac{\ddot{R}}{R}\right)$, and $\Lambda \sim \rho$, recently Ray and Mukhopadhayay (2004) have shown the equivalence of this models. They also have established a relationship between the parameters $\alpha, \beta,$ and γ of the respective models. It was mentioned that $\Lambda \sim \left(\frac{\ddot{R}}{R}\right)$ model can be viewed as a combination of $\Lambda \sim \left(\frac{\dot{R}}{R}\right)^2$ and $\Lambda \sim \dot{H}$ models (since, $\frac{\ddot{R}}{R} = \left(\frac{\dot{R}}{R}\right)^2 + \dot{H}$). Therefore, for $\dot{H} = 0$ the models $\Lambda \sim H^2$ become identical, where R(t) is the scalar factor of the Universe and $H\left(= \frac{\dot{R}}{R}\right)$ is the Hubble parameter. Since, $\Lambda \sim \left(\frac{\ddot{R}}{R}\right)$ model depends on H^2 and $\dot{H}$, and $\Lambda \sim H^2$ model has already been studied by Ray and Mukhopadhayay (2004), the point of interest is now $\Lambda \sim \dot{H}$ model. Although a number a phenomenological models have been listed by Overduin and Cooperstock (1998) (also see the references in Ray and Mukhopadhayay (2004) but model $\Lambda \sim \dot{H}$ is not included there.

This chapter is the generalizations of the work obtained earlier by Mukhopadhay and Saibal for higher dimensional space time. In this chapter we solve the Einstein field equation for the scalar factor R(t), density ρ(t) and cosmological term Λ(t), by assuming $\Lambda = \mu\dot{H}$, where μ is a parameter of $\Lambda \sim \dot{H}$, in the context of higher dimension space time. It has been shown that models is equivalent to $\Lambda \sim \left(\frac{\dot{R}}{R}\right)^2$, $\Lambda \sim \left(\frac{\ddot{R}}{R}\right)$, and $\Lambda \sim \rho$ models, when the condition for R(t), ρ(t), and Λ(t) are expressed in terms of Ω_m and Ω_Λ, respectively, they are expressed in the terms of , the matter and vaccum energy density of the universe .

The chapter is organized as follows:- Section 2.2 and 2.3 deals with Einstein field equations and their solutions. An inter-relationship of μ with $\alpha, \beta,$ and γ are presented in section 2.4. Section 2.5 deals with the physical implications and limit of the deceleration parameter for the present models equivalence of $\Lambda \sim \dot{H}$ model with $\Lambda \sim \left(\frac{\dot{R}}{R}\right)^2$, $\Lambda \sim \left(\frac{\ddot{R}}{R}\right)$ and $\Lambda \sim \rho$ models. Finally, all the results and their significance are discussed in section 2.6.

2. Einstein Field Equations

Consider the line element

$$ds^2 = dt^2 - R^2(t)(dx^2 + dy^2 + dz^2) - A^2(t)dm^2, \qquad (1)$$

where, R(t) is a scale factor and A(t) is the mass scale factor.

The Einstein field equations are given by

$$R^{ij} - \frac{1}{2}Rg^{ij} = -8\pi G\left[T^{ij} + \frac{\Lambda}{8\pi G}g^{ij}\right], \qquad (2)$$

where, $\Lambda = \Lambda(t)$ is the time dependent cosmological constant.

Also, T_j^i is the energy-momentum tensor, in the presence of perfect fluid and has the form

$$T_j^i = (p + \rho)u^i u_j - p\delta_j^i, \qquad (3)$$

where p and ρ are respectively, the energy and the pressure of the cosmic fluid, and u^i is the fluid five velocity, such that, $u^0 = 1$ and $u^i = 0$, $(i = 1,2,3,4)$ and $u^i u_i = 1$.

Hence, the Eqn. (2) yield

$$3\left[\frac{\dot{R}\dot{A}}{RA} + \frac{\dot{R}^2}{R^2}\right] = 8\pi G\rho + \Lambda, \qquad (4)$$

$$2\frac{\ddot{R}}{R} + 2\frac{\dot{R}\dot{A}}{RA} + \frac{\dot{R}^2}{R^2} + \frac{\ddot{A}}{A} = -8\pi Gp + \Lambda, \qquad (5)$$

$$3\left[\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2}\right] = -8\pi Gp + \Lambda. \qquad (6)$$

Now, put

$$\Lambda = R^m, \qquad (7)$$

in Eqns. (4), (5) and (6) we get

$$3(n+1)\frac{\dot{R}^2}{R^2} = 8\pi G\rho + \Lambda, \qquad (8)$$

$$(n+2)\frac{\ddot{R}}{R} + (n^2+n+1)\frac{\dot{R}^2}{R^2} = -8\pi Gp + \Lambda, \qquad (9)$$

$$3(n+1)\frac{\ddot{R}}{R} = -8\pi G[\rho + p(n+1)] + \Lambda n, \qquad (10)$$

where, over dot indicates a derivative with respect to time t.

Let us now consider the barotropic equation of state in the form

$$p = \omega\rho \, , \qquad (11)$$

where ω, the equation of state parameter, for the dust, radiation, vacuum fluid and stiff fluid can take the constant values 0, $1/3$, -1,+1 respectively.

Let us assume that G does not vary with space and time. Then we assume

$$\Lambda = \mu\dot{H}, \qquad (12)$$

where, μ is a free parameter, we get from the Eqns. (8) and (10) the following two modified equations

$$4\pi G\rho = \frac{1}{2}[3(n+1)H^2 + \mu\dot{H}], \qquad (13)$$

and $$3(n+1)(H^2+\dot{H}) = -8\pi G\rho[1+\omega(n+1)] + n\mu\dot{H}. \qquad (14)$$

Eqns. (13) and (14) on simplification, yield the differential equation

$$(3-\mu-\mu\omega)\dot{H} = -3[2+\omega(n+1)]H^2. \qquad (15)$$

2.3 The Models: Now Eqn. (15) on integration gives

$$H = \frac{3-\mu-\mu\omega}{3[2+\omega(n+1)]t}. \qquad (16)$$

Putting $H = \frac{\dot{R}}{R}$ in Eqn. (16) and integrating it further we get our general solution set as

$$R(t) = Ct^{\frac{3-\mu-\mu\omega}{3[2+\omega(n+1)]}}, \qquad (17)$$

$$\rho(t) = \frac{(3-\mu-\mu\omega)[(3-\mu)(n+1)+2\mu]}{24\pi G[2+\omega(n+1)]^2}t^{-2}, \qquad (18)$$

$$\Lambda(t) = \frac{-\mu(3-\mu-\mu\omega)}{3[2+\omega(n+1)]}t^{-2}, \qquad (19)$$

where C is an integration constant.

Case (i) Dust case $(\omega = 0)$:

For dust case, Eqns. (17), (18), (19) and (16) respectively takes the form

$$R(t) = Ct^{(3-\mu)/6}, \tag{20}$$

$$\rho(t) = \frac{(3-\mu)[(3-\mu)(n+1)+2\mu]}{96\pi G} t^{-2}, \tag{21}$$

$$\Lambda(t) = \frac{-\mu(3-\mu)}{6} t^{-2}, \tag{22}$$

$$t = \frac{3-\mu}{6H}. \tag{23}$$

Eqn. (2.23) suggests that for physically valid ρ $(i.e.\ \rho > 0)$, we should get $\mu < 3$. So μ can be negative as well. Again, from Eqn. (2.24) we find that for a repulsive Λ, the constraint on μ is that it must be negative. Thus, Eqns. (2.23), (2.24) and (2.25) all points towards a negative μ.

Case (ii) Radiation case $(\omega = 1/3)$:

For radiation case, Eqns. (2.18), (2.19), (2.20) and (2.17) respectively takes the form

$$R(t) = Ct^{\frac{9-4\mu}{3(n+1)}}, \tag{2.26}$$

$$\rho(t) = \frac{(9-\mu-3\mu)[(3-\mu)(n+1)+2\mu]}{8\pi G(n+1)^2 t^2}, \tag{2.27}$$

$$\Lambda(t) = \frac{-\mu(9-\mu-3\mu)}{3(n+1)} t^{-2}, \tag{2.28}$$

$$t = \frac{(9-\mu-3\mu)}{3(n+1)H}. \tag{2.29}$$

Eqn. (2.27) suggests that for physically valid ρ, $\mu < 9/4$ whereas Eqn. (2.28) demands a negative μ for repulsive Λ. Thus, in this case also a negative μ is necessary for Eqns. (2.27) - (2.29).

Using the value of t from Eqn. (2.25) in (2.26) we get

$$\Omega_m = 1 + \frac{2\mu}{(n+1)(3-\mu)}, \tag{2.30}$$

where, $\Omega_m \left(= \frac{8\pi G\rho}{3(n+1)H^2}\right)$ is the matter-energy density of the Universe.

Again, using Eqn. (2.25) we get from Eqn. (2.24)

$$\Omega_\Lambda = -\frac{2\mu}{(n+1)(3-\mu)}, \qquad (2.31)$$

where, $\Omega_\Lambda \left(= -\frac{\Lambda}{3(n+1)H^2}\right)$ is the vacuum-energy density of the Universe.

Adding Eqns. (2.30) and (2.31) we get

$$\Omega_m + \Omega_\Lambda = 1, \qquad (2.32)$$

which is another form of Eqn. (2.2) for flat Universe.

Also, using the value of μ from Eqn. (2.30) we get from Eqn. (2.25)

$$t = \frac{1}{[\Omega_m(n+1)-(n-1)]H}. \qquad (2.33)$$

Thus, if t_0 and H_0 are the values of t and H at the present epoch, then

$$t_0 = \frac{1}{[\Omega_{m0}(n+1)-(n-1)]H_0}. \qquad (2.34)$$

2.3.1 Three different forms of Λ

(i) Model for $\Lambda \sim \left(\frac{\dot{R}}{R}\right)^2$:-

If we use $\Lambda = \alpha\left(\frac{\dot{R}}{R}\right)^2 = \alpha H^2$, where α is a constant, then Eqn. (2.12) becomes

$$3(n+1)\frac{\dot{R}^2}{R^2} = 8\pi G\rho[1+\omega(n+1)] + n\alpha\frac{\dot{R}^2}{R^2}, \qquad (2.35)$$

$$\frac{\ddot{R}}{R} + \frac{1}{3(n+1)}\{[3(n+1)-\alpha][1+\omega(n+1)] - n\alpha\}\dot{R}^2 = 0. \qquad (2.36)$$

Integrating Eqn. (2.36), we get

$$\dot{R}R^{\frac{\{[3(n+1)-\alpha][1+\omega(n+1)]-n\alpha\}}{3(n+1)}} = k_1, \qquad (2.37)$$

where, k_1 is an integrating constant.

Now, on integrating above Eqn. (2.36) we get

$$R(t) = \left|\frac{(6-\alpha)+\omega[3(n+1)-\alpha]}{3}k_1 t\right|^{\frac{3}{(6-\alpha)+\omega[3(n+1)-\alpha]}}. \qquad (2.38)$$

Now, from Eqn. (2.10), we have

$$\rho(t) = \frac{1}{8\pi G}[3(n+1)-\alpha]\frac{\dot{R}^2}{R^2}, \qquad (2.39)$$

$$\rho(t) = \frac{9}{8\pi G}\frac{[3(n+1)-\alpha]}{[(6-\alpha)+\omega[3(n+1)-\alpha]]^2 t^2}, \qquad (2.40)$$

$$\Lambda(t) = \alpha\frac{\dot{R}^2}{R^2} = \frac{9\alpha}{[(6-\alpha)+\omega[3(n+1)-\alpha]]^2 t^2}, \qquad (2.41)$$

$$t = \frac{3}{[(6-\alpha)+\omega[3(n+1)-\alpha]]H}. \qquad (2.42)$$

(ii) Model for $\Lambda \sim \left(\frac{\ddot{R}}{R}\right)$:-

If we consider $\Lambda = \beta\left(\frac{\ddot{R}}{R}\right)$, where β is a constant, then Eqn. (2.12) reduces to

$$R\ddot{R} + \frac{3[1+\omega(n+1)]}{(3-\beta-\beta\omega)}\dot{R}^2 = 0. \qquad (2.43)$$

Integrating above Eqn. (2.43) we get

$$R(t) = \left[\frac{6+3\omega(n+1)-\beta(1+\omega)}{(3-\beta-\beta\omega)}k_2 t\right]^{\frac{(3-\beta-\beta\omega)}{6+3\omega(n+1)-\beta(1+\omega)}}, \qquad (2.44)$$

where, k_2 is an integrating constant.

Now, from Eqn. (2.10) we obtain

$$\rho(t) = \frac{3}{8\pi G}\frac{(3-\beta-\beta\omega)}{[6+3\omega(n+1)-\beta(1+\omega)]^2}\left[\frac{(3-\beta-\beta\omega)(n+1)+\beta[1+\omega(n+1)]}{t^2}\right], \qquad (2.45)$$

$$\Lambda(t) = \beta\frac{\ddot{R}}{R} = \frac{-3\beta(3-\beta-\beta\omega)[1+\omega(n+1)]}{[6+3\omega(n+1)-\beta(1+\omega)]^2 t^2}, \qquad (2.46)$$

$$t = \frac{(3-\beta-\beta\omega)}{[6+3\omega(n+1)-\beta(1+\omega)]H}. \qquad (2.47)$$

(iii) Model for $\Lambda \sim \rho$:-

If we set $\Lambda = 8\pi G\gamma\rho$, where γ is a free parameter, then using Eqn. (2.10), Eqn. (2.12) reduces to

$$3(n+1)\frac{\ddot{R}}{R} = -\frac{3(n+1)}{(\gamma+1)}[1+\omega(n+1)-n\gamma]\frac{\dot{R}^2}{R^2}, \qquad (2.48)$$

or

$$R\ddot{R} + \frac{[1+\omega(n+1)-n\gamma]}{(\gamma+1)}\dot{R}^2 = 0. \qquad (2.49)$$

Integrating above Eqn. (2.49) we get

$$R(t) = \left[\frac{2+\omega(n+1)+\gamma(1-n)}{(\gamma+1)} k_3 t\right]^{\frac{(\gamma+1)}{2+\omega(n+1)+\gamma(1-n)}}, \qquad (2.50)$$

where, k_3 is an integrating constant.

Now, from Eqn. (2.10) we obtain

$$\rho(t) = \frac{3(n+1)(\gamma+1)}{8\pi G[2+\omega(n+1)+\gamma(1-n)]^2 t^2}, \qquad (2.51)$$

$$\Lambda(t) = 8\pi G\gamma\rho = \frac{3(n+1)(\gamma+1)\gamma}{[2+\omega(n+1)+\gamma(1-n)]^2 t^2}, \qquad (2.52)$$

$$t = \frac{(\gamma+1)}{[2+\omega(n+1)+\gamma(1-n)]H}. \qquad (2.53)$$

2.4 Equivalent Relationship Between Λ-Dependent Models

Now, let us find out the inter-relation between $\alpha, \beta,$ and γ and hence the equivalence of the different forms of the dynamical cosmological terms $\Lambda \sim \left(\frac{\dot{R}}{R}\right)^2$, $\Lambda \sim \left(\frac{\ddot{R}}{R}\right)$ and $\Lambda \sim \rho$.

Using Eqn. (2.42) in Eqn. (2.40) in the definition of the cosmic matter-density parameter $\Omega_m = \frac{8\pi G\rho}{3(n+1)H^2}$, we get

$$\Omega_{m\alpha} = 1 - \frac{\alpha}{3(n+1)}, \qquad (2.54)$$

where, $\Omega_{m\alpha}$ is the cosmic matter-density parameter for the α- related dynamic Λ-model.

Using Eqn. (2.42) in Eqn. (2.41) in the definition of the cosmic vacuum-density parameter $\Omega_\Lambda = \frac{\Lambda}{3(n+1)H^2}$, we have

$$\Omega_{\Lambda\alpha} = \frac{\alpha}{3(n+1)}, \qquad (2.55)$$

where, in the similar fashion, $\Omega_{\Lambda\alpha}$ is the cosmic vacuum-density parameter for the α-related dynamic Λ-model. From Eqns. (2.54) and (2.55) we obtain

$$\Omega_{m\alpha} + \Omega_{\Lambda\alpha} = 1, \qquad (2.56)$$

which is the relation between the cosmic matter and vacuum-density parameters.

Similarly, from Eqns. (2.45), (2.46) and (2.47) we obtain

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$$\Omega_{m\beta} = 1 + \frac{\beta}{(3-\beta)(n+1)}, \qquad (2.57)$$

$$\Omega_{\Lambda\beta} = -\frac{\beta}{(3-\beta)(n+1)}, \qquad (2.58)$$

where, $\Omega_{m\beta}$ and $\Omega_{\Lambda\beta}$ are respectively the cosmic matter and energy density parameters for the β- related dynamic Λ-model.

Adding Eqns. (2.57) and (2.58) we get

$$\Omega_{m\beta} + \Omega_{\Lambda\beta} = 1, \qquad (2.59)$$

which is again the relation between the cosmic matter and vacuum-density parameters.

In the same manner from Eqns. (2.51) – (2.53) to the cosmic matter and vacuum energy density parameters, $\Omega_{m\gamma}$ and $\Omega_{\Lambda\gamma}$ are respectively for the model $\Lambda \sim \rho$ also satisfy the relation

$$\Omega_{m\gamma} + \Omega_{\Lambda\gamma} = 1, \qquad (2.60)$$

where, $\gamma = \frac{\Omega_{\Lambda\gamma}}{\Omega_{m\gamma}}$.

Thus from relations (2.56), (2.59) and (2.60) without loss of generality, we can set

$$\Omega_{m\alpha} = \Omega_{m\beta} = \Omega_{m\gamma} = \Omega_m, \qquad (2.61)$$

$$\Omega_{\Lambda\alpha} = \Omega_{\Lambda\beta} = \Omega_{\Lambda\gamma} = \Omega_\Lambda, \qquad (2.62)$$

where, Ω_m and Ω_Λ are respectively the cosmic matter and vacuum energy density parameters which in absence of any curvature satisfy the general relation $\Omega = \Omega_m + \Omega_\Lambda = 1$.

2.4.1 Equivalence of three forms of Λ

By the above relation, we get

$$\frac{\alpha}{3(n+1)} = -\frac{\beta}{(3-\beta)(n+1)} = -\frac{2\mu}{(n+1)(3-\mu)} = \frac{\gamma}{\gamma+1}, \qquad (2.63)$$

or

$$\alpha = \frac{3\beta}{\beta-3} = \frac{6\mu}{\mu-3} = \frac{3(n+1)\gamma}{\gamma+1}, \qquad (2.64)$$

2.5 Physical Features of the Models

The deceleration parameter (q) is defined as

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -\left(1 + \frac{\dot{H}}{H^2}\right). \qquad (2.65)$$

Thus, using Eqn. (2.18) we have

$$q = -\left[1 - \frac{3\{2+\omega(n+1)\}}{3-\mu-\mu\omega}\right]. \qquad (2.66)$$

For an accelerating universe $q < 0$ and hence

$$1 - \frac{3\{2+\omega(n+1)\}}{3-\mu-\mu\omega} > 0,$$

$$\mu < \frac{-3\{1+\omega(n+1)\}}{1+\omega}. \qquad (2.67)$$

Eqn. (2.67) tells us that for a dust-filled accelerating universe, μ should be less than -1. We have already shown that for physically valid ρ, Λ and t, μ must be negative.

2.6 Conclusion

In the chapter, by choosing phenomenological model of Λ, $\Lambda \sim \dot{H}$, it has been shown that this model of Λ is equivalent to three types of Λ, $\Lambda \sim \left(\frac{\dot{R}}{R}\right)^2$, $\Lambda \sim \left(\frac{\ddot{R}}{R}\right)$, and $\Lambda \sim \rho$. A relationship is established between the parameter in both dust and radiation cases. Since $\frac{\ddot{R}}{R} = \frac{H^2}{H}$, it is clear that the dependency of the parameter ω and β is due to $\Lambda \sim \dot{H}$ part because a relation of μ with Ω_m and Ω_Λ also contains ω and hence μ behaves differently with cosmic matter and vacuum energy parameter in the dust and radiation cases.

Finally, it should be mentioned that any linear combination of $\Lambda \sim \left(\frac{\dot{R}}{R}\right)^2$, $\Lambda \sim \left(\frac{\ddot{R}}{R}\right)$ and $\Lambda \sim \rho$ is equivalent.

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